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THE LAMINAR BOUNDARY LAYER IN A RADIATING-ABSORBING GAS NEAR A FLAT PLATE

V. P. Zamurayev (Novosibirsk)

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The steady-state problem of the laminar flow of a radiating-absorbing gas in the boundary layer near a flat plate has been considered in several papers [1-6]. However, due to the assumptions made by their authors as regards the nature of the radiation, the applicability of the solutions obtained therein is limited. The degree of exactness of the methods they employed in specifying the radiation is moreover uncertain. */73

In the present paper we shall be concerned with the laminar boundary layer near a plate. We intend to use a more exact description of radiation transfer. Heat transfer takes place by ordinary heat conduction and radiation. A number of simplifying assumptions already used in the aforementioned papers are made about the radiation. Thus, we apply the hypothesis of local thermodynamic equilibrium according to which the radiating power and coefficient of absorption are related by Kirchhoff's law. The medium is assumed to be gray. The radiation flux along the plate is neglected in comparison with the radiation flux across the plate. Such an assumption is valid provided the temperature variation along the plate over the length of the radiation path is small. The effect of but slight temperature variation along the plate is to render the radiation flux across the plate determinable by the temperature profile in the given section. The wall is assumed to be absolutely black. The physical properties of the medium may be temperature dependent.

The asymptotic behavior of heat transfer far away from the tip of the plate is considered. A difference method of solving a system of partial differential equations with a complex integrodifferential energy equation is used to solve the problem over the entire length of the plate. In its basic features, this method is similar to the difference method used to solve boundary layer equations without regard for radiation [7-9]. The paper concludes with a discussion of the results

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*/Numbers in the margin indicate pagination of the original foreign text.

of computing one of the cases considered in [1]. The character of heat transfer and the possibility of specifying radiation in the radiant heat conduction approximation are investigated.

1. We consider the steady-state laminar boundary layer near a flat plate.

In the variables x (the longitudinal coordinate) and y (the transverse coordinate) the problem posed is described by the system of equations

$$\begin{gathered}\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0\\ \rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y}\\ \rho u c_p\frac{\partial T}{\partial x}+\rho v c_p\frac{\partial T}{\partial y}=-\frac{\partial q}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2\end{gathered} \tag{1.1}$$

where u and v are the components of the velocity along and across the plate, T the temperature, ρ the density, μ the coefficient of viscosity, c_p the specific heat capacity at constant pressure, and q total heat flux along y, equal to the sum of fluxes due to ordinary heat conduction q_m and to radiation q_r,

$$q=q_m+q_r \tag{1.2}$$

The heat flux q_m is defined in the usual way,

$$q_m=-\lambda\,\partial T/\partial y \tag{1.3}$$

where λ is the thermal conductivity coefficient.

The heat flux due to radiation q_r can be found by using its expression /74 obtained by integrating over the spectrum and solid angle the intensity of radiation in the form of the formal solution of the transfer equation multiplied by the cosine of the angle between the direction of radiation and the y-axis, with due allowance for the assumption of grayness of the medium and no regard for the temperature variation along x over several radiation path lengths,

$$q_r=2\sigma T_0^4E_3(\tau)+\int_0^\tau 2\sigma T^4E_2(\tau-t)\,dt-\int_\tau^\infty 2\sigma T^4E_2(t-\tau)\,dt \tag{1.4}$$

Here σ is the Stefan-Boltzmann constant, $E_n(\tau)$ the integral exponential function, and τ the optical thickness of the gas layer given by the equation (ι is the radiation path length)

$$\tau = \int_0^y \frac{dy}{l} \tag{1.5}$$

The subscript 0 denotes the values of the parameters at the wall. The quantities ρ, μ, c_p, λ, ι are generally functions of temperature.

Instead of the equation of discontinuity we can consider the two equivalent equations (ψ is the stream function)

$$\rho u = \frac{\partial \psi}{\partial y}, \qquad \rho v = -\frac{\partial \psi}{\partial x} \tag{1.6}$$

The solution of system (1.1)-(1.5) will be sought in the region $x > 0$, $y \geq 0$ under the boundary conditions

$$u = \nu = 0, \quad T = T_0 \text{ for } y = 0; \qquad u = u_\infty, \quad T = T_\infty \text{ for } y = \infty \tag{1.7}$$

2. The problem just formulated is solved numerically. The tip $x = y = 0$ is a special point (in the neighborhood of this point the friction stress and heat flux due to ordinary heat conduction behave as $1/\sqrt{x}$), so that in order to derive a single algorithm for numerical computation valid in the entire region we introduce the new independent variables

$$\xi = \frac{2\sigma T_1^4}{\rho_1 u_\infty c_{p1} T_1} \frac{x}{l_1}, \qquad \eta = \left(\frac{\rho_1 u_\infty}{\mu_1 x}\right)^{1/2} \int_0^y \frac{\rho}{\rho_1} dy \tag{2.1}$$

and replace the stream function ψ (x, y) by the new function f (ξ, η),

$$\psi = \sqrt{\rho_1 u_\infty \mu_1 x}\, f \tag{2.2}$$

As a result, all of the derivatives appearing in the conservation equations become finite, and the functions f, u, and T which we are seeking in the new variables, vary weakly along the plate so that computation can be effected with a larger interval along ξ without loss of accuracy.

The boundary conditions for $y = \infty$ must be considered at some finite distance from the plate where the functions u and T begin to differ from their limiting values u_∞ and T_∞ by an amount not exceeding the difference scheme error. In the new variables the thickness of the boundary layer varies weakly and the solution may be sought in the standard region $\xi \geq 0$, $0 \leq \eta \leq \eta_\infty$.

Let us introduce the dimensionless quantities

$$u' = \frac{u}{u_\infty},\ T' = \frac{T}{T_1},\ \rho' = \frac{\rho}{\rho_1},\ \mu' = \frac{\mu}{\mu_1},\ c_p' = \frac{c_p}{c_{p1}},\ \lambda' = \frac{\lambda}{\lambda_1},\ l' = \frac{l}{l_1},\ q' = \frac{q}{\sigma T_1^4} \tag{2.3}$$

where the physical parameters accompanied by the subscript 1 signify their values at the temperature T_1. System (1.1)-(1.6) with boundary conditions (1.7) in terms of dimensionless quantities may be written as /75

$$u' = \frac{\partial f}{\partial \eta} \tag{2.4}$$

$$\frac{\partial}{\partial \eta}\rho'\mu'\frac{\partial u'}{\partial \eta} + \frac{f}{2}\frac{\partial u'}{\partial \eta} + \xi\left(\frac{\partial f}{\partial \xi}\frac{\partial u'}{\partial \eta} - u'\frac{\partial u'}{\partial \xi}\right) = 0 \tag{2.5}$$

$$\frac{1}{P_1}\frac{\partial}{\partial \eta}\rho'\lambda'\frac{\partial T'}{\partial \eta} + \frac{f}{2}c_p'\frac{\partial T'}{\partial \eta} + \xi c_p'\left(\frac{\partial f}{\partial \xi}\frac{\partial T'}{\partial \eta} - u'\frac{\partial T'}{\partial \xi}\right) + \xi\Phi + \varepsilon_1\rho'\mu'\left(\frac{\partial u'}{\partial \eta}\right)^2 = 0 \tag{2.6}$$

$$\Phi = -\frac{1}{2\sqrt{\xi\varepsilon}}\frac{\partial q_r'}{\partial \eta}, \quad \rho' l'\Phi = (\theta_0^4 - T'^4)E_2(\tau) +$$
$$+ (\theta_\infty^4 - T'^4)E_2(\tau_\infty - \tau) + \int_0^{\tau_\infty}[T'^4(t) - T'^4]E_1(|\tau - t|)\,dt \tag{2.7}$$

$$\tau = \sqrt{\varepsilon\xi}\int_0^\eta \frac{d\eta}{\rho' l'} \tag{2.8}$$

The boundary conditions are

$$f = u' = 0,\ T' = \theta_0 \text{ for } \eta = 0, \qquad u' = 1,\ T' = \theta_\infty \text{ for } \eta = \eta_\infty \tag{2.9}$$

where P_1 is the Prandtl number,

$$P_1 = \frac{c_{p_1}\mu_1}{\lambda_1}, \quad \varepsilon = \frac{\mu_1 c_{p_1} T_1}{2\sigma T_1^4 l_1}, \quad \varepsilon_1 = \frac{u_\infty^2}{c_{p1}T_1} \tag{2.10}$$

The expression for Φ was obtained allowing for the fact that $T' = \theta_\infty$ for $\eta \geq \eta_\infty$. The heat fluxes can be found from the formulas

$$q_m' = -\frac{2\sqrt{\varepsilon}}{P_1\sqrt{\xi}}\rho'\lambda'\frac{\partial T'}{\partial \eta} \tag{2.11}$$

$$q_r' = 2\theta_0^4 E_3(\tau) - 2\theta_\infty^4 E_3(\tau_\infty - \tau) + 2\int_0^\tau T'^4 E_2(\tau - t)\,dt -$$
$$-2\int_\tau^{\tau_\infty} T'^4 E_2(t - \tau)\,dt \tag{2.12}$$

3. From equation (2.8) we see that with small values of ξ the boundary layer is optically transparent and that the smaller the value of ε the longer it remains transparent. The term related to radiation in energy equation (2.6) is the product of ξ by the bounded function Φ ; with small ξ it is small in comparison with the other terms of the equation. Hence, the effect of radiation

is small. As $\xi \to 0$ it disappears entirely. The temperature profile tends to a self-similar profile in the absence of radiation. However, the singularity at $\xi = 0$ never vanishes entirely. Being bounded, the function Φ has unbounded derivatives with respect to ξ. The first of its derivatives behaves as $\xi^{-\frac{1}{2}} \cdot \ln \xi$. This is due to the fact that the optical thickness of the boundary layer increases as $\sqrt{\xi}$ (for small ξ). As a result, the function Φ varies very abruptly.

4. Integrating by parts in the right side of equation (1.4) and applying (1.5), we obtain an expression for q_r in dimensionless form,

$$q_r' = -\frac{4}{3}\frac{1}{\sqrt{\varepsilon\xi}}\frac{\partial T'^4}{\partial \alpha} + \frac{1}{\sqrt{\varepsilon\xi}}\frac{\partial T'^4}{\partial \alpha}\Big|_{\alpha=0} 2E_4(\sqrt{\varepsilon\xi}\alpha) + \qquad \left(\alpha = \int_0^\eta \frac{d\eta}{\rho' l'}\right)$$
$$+ \frac{2}{\sqrt{\varepsilon\xi}}\int_0^\alpha \frac{\partial^2 T'^4}{\partial t^2} E_4\left[\sqrt{\varepsilon\xi}(\alpha - t)\right] dt - \frac{2}{\sqrt{\varepsilon\xi}}\int_\alpha^\infty \frac{\partial^2 T'^4}{\partial t^2} E_4\left[\sqrt{\varepsilon\xi}(t - \alpha)\right] dt \qquad (4.1)$$

If the quantity $\sqrt{\varepsilon\xi} \to 0$, then the integrands in (4.1) tend to zero /76 everywhere as $(\sqrt{\varepsilon\xi}\,|\alpha - t|)^{-1} \exp(-\sqrt{\varepsilon\xi}\,|\alpha - t|)$ with exception of the neighborhood of the point $t = \alpha$ tending to zero, where they are finite. As a result, the corresponding terms are of the order $(\varepsilon\xi)^{-1}$. Therefore, with $\sqrt{\varepsilon\xi} \gg \alpha^{-1}$ the expression

$$q_r' = -\frac{4}{3}\frac{1}{\sqrt{\varepsilon\xi}} [illegible] \text{ or } [illegible] \qquad (4.2)$$

may be used to approximate q_r' instead of (2.12).

Thus, with large values $\varepsilon\xi$ the radiation flux outside the wall is obtained to within the approximation of radiant heat conduction.

The situation near the wall in a layer of thickness measuring several radiation path lengths is somewhat different (the thickness of such a layer in the variables η, ξ tends to zero as $1/\sqrt{\varepsilon\xi}$). Since L is the characteristic dimension of the problem as $1/L \to 0$ (so that $1/\sqrt{\varepsilon\xi} \to 0$), we obtain the following expression for the radiation flux in this layer:

$$[illegible]\left[1 - \frac{3}{2}E [illegible]\right] [illegible] \frac{[illegible]}{\partial\eta} \qquad (4.3)$$

This expression differs from (4.2) by the factor $[1 - \frac{3}{2}E_4(\tau)]$, which varies from 0.5 at the wall to 1 in the gas stream (for $\tau = 4$ its deviation from unity is less than 0.4%). The ratio of heat fluxes is therefore given by the expression

$$\frac{q_r}{q_m} = \frac{16}{3}\left[1 - \frac{3}{2}E_4(\tau)\right]\frac{\sigma T^3 l}{\lambda}$$

Thus, with small 1/L near a wall in a gas layer of a thickness measuring several radiation path lengths there occurs a redistribution of the heat transferred by radiation and ordinary heat conduction in the direction of increasing molecular heat flux, and a corresponding reduction of the radiation flux (the total heat flux remains almost constant). This takes place as a result of a more abrupt temperature drop toward the wall. The relative thickness of this layer is small, however, so that the correct total heat flux can be obtained by considering radiation in the radiant heat conduction approximation. The radiation component of the heat flux will be exaggerated at least two-fold as the parameter 1/L goes from zero to infinity. In [1] this drawback of the radiant heat conduction approximation is at least partly removed by introducing a thin layer next to the wall where the coefficient in the expression for the radiation heat flux is one half of that in the remaining region.

5. System (2.4)-(2.8) with boundary conditions (2.9) will be solved by the method of finite differences.

The flow region $\xi \geq 0$, $0 \leq \eta \leq \eta_\infty$ = const is broken down into characteristic strips of width h. We consider the arithmetic means of the required function f_p (f_p = f, u', T') on the left (i - 1)-th and right i-th boundaries of the strip

$$f_p^\circ = \tfrac{1}{2}(f_{p,\,i-1} + f_{pi}) \tag{5.1}$$

where the subscript i means that the function is taken for an ξ equal to ξ_i = ih (i = 0, 1, 2,....).

These mean values differ from the exact values on the mid-line of the strip by an amount on the order of h^2

System (2.4)-(2.6) with boundary conditions (2.9) is written out on the mid-line of the characteristic strip; the exact values are replaced by the mean values f_p^o, and the derivatives with respect to ξ by the difference analog

$$\frac{\partial f_p}{\partial \xi} \rightarrow \frac{f_p^\circ - f_{p,\,i-1}}{h/2} \tag{5.2}$$

In order to avoid iteration, which increases computer time considerably if radiation is present, and also to prevent an increase in the system approximation error, the quantity Φ describing the radiation, as well as the quantities $\rho'\mu'$, $\rho'\lambda'$, c_p' and the velocity u' in (2.4) — each of these quantities is replaced by a linear combination of its values on the boundaries of the preceding /77 strip, so that the system approximation error remains on the order of h^2,

$$f_q \to f_q^o = (1 + {}^1/_2 b)\, f_{q,i-1} - {}^1/_2 b\, f_{q,i-2} \quad (b = h_2/h_1) \tag{5.3}$$

where h_1 is the preceding interval, and h_2 is the new interval.

As a result, the system of linear partial differential equations (2.4)-(2.6) is reduced to a system of linear ordinary differential equations

$$\frac{df^o}{d\eta} = u^o \tag{5.4}$$

$$\frac{d}{d\eta}(\rho\mu)^o \frac{du^o}{d\eta} + \frac{f^o}{2}\frac{du^o}{d\eta} + \xi^o\left(\frac{f^o - f_{i-1}}{h/2}\frac{du^o}{d\eta} - u^o\frac{u^o - u_{i-1}}{h/2}\right) = 0 \tag{5.5}$$

$$\frac{1}{P_r}\frac{d}{d\eta}(\rho\lambda)^o \frac{dT^o}{d\eta} + \frac{f^o}{2}c_p^o\frac{dT^o}{d\eta} + \xi^o c_p^o\left(\frac{f^o - f_{i-1}}{h/2}\frac{dT^o}{d\eta} - u^o\frac{T^o - T_{i-1}}{h/2}\right) + \xi^o\Phi^o + e_1(\rho\mu)^o\left(\frac{du^o}{d\eta}\right)^2 = 0 \tag{5.6}$$

The boundary conditions are

$$f^o = u^o = 0,\ T^o = \theta_0^o \text{ for } \eta = 0;\ u^o = 1,\ T^o = \theta_\infty^o \text{ for } \eta = \eta_\infty \tag{5.7}$$

This system is solved in the following sequence of steps. First, the function $f^o(\eta)$ is found from equation (5.4). Next, equation (5.5) is solved for $u^o(\eta)$. Finally, the function $T^o(\eta)$ is found from equation (5.6).

Once system (5.4)-(5.6) has been solved, the algebraic relations $f_{pi} = 2f_p^o - f_{p,i-1}$ implied by (5.1) are used to find the values of the required functions f, u', and T' for $\xi = \xi_i = ih$ to within an error on the order of h^2.

To solve the linear system of ordinary differential equations (5.4)-(5.6) the flow region in the plane ξ, η is broken down into n horizontal strips of width $\Delta = \eta_\infty/n$, and each equation of (5.4)-(5.6) is approximated by a difference equation of second-order exactness. The values of the function $f^o(\eta)$ for $\eta = j\Delta$ are then found from the formula

$$f_j^o = f_{j-1}^o + \frac{\Delta}{2}(u_{j-1}^o + u_j^o), \quad f_0^o = 0 \quad (j = 1, 2, \ldots, n-1)$$

Instead of equations (5.5) and (5.6) we make use of the corresponding difference equations

$$
\begin{aligned}
&[(\rho\mu)_j'' + (\rho\mu)_{j+1}'' + \frac{\Delta}{2} f_j^{\circ} + 2\xi^{\circ} \frac{\Delta}{h}(f_j^{\circ} - f_{i-1j})]\, u_{j+1}^{\circ} - [(\rho\mu)_{j-1}'' + 2(\rho\mu)_j'' + \\
&\quad + (\rho\mu)_{j+1}'' + 4\xi^{\circ} \frac{\Delta^2}{h} u_j'']\, u_j^{\circ} + [(\rho\mu)_{j-1}'' + (\rho\mu)_j'' - \frac{\Delta}{2} f_j^{\circ} - \\
&\quad - 2\xi^{\circ} \frac{\Delta}{h}(f_j^{\circ} - f_{i-1j})]\, u_{j-1}^{\circ} + 4\xi^{\circ} \frac{\Delta^2}{h} u_j'' u_{i-1j}' = 0
\end{aligned}
$$

$$
\begin{aligned}
&[(\rho\lambda)_j'' + (\rho\lambda)_{j+1}'' + \frac{\Delta}{2} P_1 f_j^{\circ} c_{pj}'' + 2\xi^{\circ} \frac{\Delta}{h} P_1 c_{pj}'' (f_j^{\circ} - f_{i-1j})]\, T_{j+1}^{\circ} - [(\rho\lambda)_{j-1}'' + \\
&+ 2(\rho\lambda)_j'' + (\rho\lambda)_{j+1}'' + 4\xi^{\circ} \frac{\Delta^2}{h} P_1 c_{pj}'' u_j^{\circ}]\, T_j^{\circ} + [(\rho\lambda)_{j-1}'' + (\rho\lambda)_j'' - \frac{\Delta}{2} P_1 f_j^{\circ} c_{pj}'' - \\
&- 2\xi^{\circ} \frac{\Delta}{h} P_1 c_{pj}'' (f_j^{\circ} - f_{i-1j})]\, T_{j-1}^{\circ} + 4\xi^{\circ} \frac{\Delta^2}{h} P_1 c_{pj}'' u_j^{\circ} T'_{i-1j} + 2\xi^{\circ} \Delta^2 P_1 \Phi_j'' + \\
&\qquad + \frac{\varepsilon_1}{2} P_1 (\rho\mu)_j'' (u_{j+1}^{\circ} - u_{j-1}^{\circ})^2 = 0
\end{aligned}
$$

$$u_0^{\circ} = 0, \quad u_n^{\circ} = 1; \qquad T_0^{\circ} = \theta_0^{\circ}, \quad T_n^{\circ} = \theta_{\infty}^{\circ}$$

Each of the above is a second-order equation with known values of the functions sought at both ends of the interval. They are solved by the sweep method.

The quantity $(\rho' l' \Phi)_{ij}$ is computed from the formula /78

$$
(\rho' l' \Phi)_{ij} = (\theta_0^4 - T_{ij}'^4) E_3(\tau_{ij}) + (\theta_\infty^4 - T_{ij}'^4) E_3(\tau_{\infty i} - \tau_{ij}) + \sqrt{\varepsilon \xi_i}\, \frac{\Delta}{2} \sum_{k=1}^{n} (F_{ik-1j} + F_{ikj})
$$

where

$$
F_{ikj} = (T'^4_{ik} - T'^4_{ij}) E_1(|\tau_{ik} - \tau_{ij}|) \frac{1}{(\rho' l')_{ik}}
$$

The optical thickness τ_{ij} is computed from the difference analog of equation (2.8),

$$
\tau_{ij} = \tau_{ij-1} + \sqrt{\varepsilon \xi_i}\, \frac{\Delta}{2} \left[\frac{1}{(\rho' l')_{ij-1}} + \frac{1}{(\rho' l')_{ij}} \right] \quad (j = 1, 2, \ldots, n)
$$

$$\tau_{i0} = 0$$

In order to begin computation it is necessary to know the velocity and temperature profiles in the initial ($\xi = 0$) and first ($\xi = h$) sections. In these sections the solution is sought by the successive approximation method.

6. The results of calculations are given for the case $\theta_\infty = 1$, $\theta_0 = 0.1$, $\varepsilon = 0.2$, $P_1 = 1$, $\varepsilon_1 = 0$. The physical properties in this case were set constant

-- the quantities $\rho'\mu'$, $\rho'\lambda'$, c_p', $\rho'\iota'$ were equated to unity. All of these conditions correspond exactly to one of the variants considered in [1].

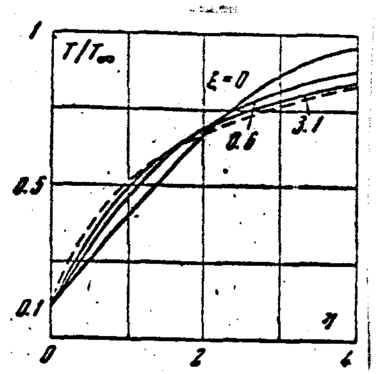

Figure 1.

The problem was computed with an interval variable along ξ. The accuracy was checked by repeated computation with other intervals both along ξ and along η. The initial computation was carried out for the following values of the interval along ξ:

$$h = 0.002 \text{ for } 0 \leq \xi \leq 0.008$$

$$h = 0.004 \text{ for } 0.008 \leq \xi \leq 0.02$$

$$h = 0.008 \text{ for } 0.02 \leq \xi \leq 0.06$$

$$h = 0.02 \text{ for } 0.06 \leq \xi \leq 0.3$$

$$h = 0.1 \text{ for } 0.3 \leq \xi \leq 5.1$$

The interval along η in this case was 0.4, and the value of η on the outer boundary was 16. Computation on a computer required 5 minutes.

Repeated computation was carried out up to $\xi = 3.1$ for intervals half as great both along ξ and along η. The value of η_∞ up to $\xi = 0.5$ was set equal to 13 and then increased to 16.

The maximum difference between the first and second computed values of the temperature was 0.3%; the corresponding difference in the values of heat fluxes to the wall was 0.7% (for $\xi = 3.1$). The difference in the values of Φ was as high as several percent.

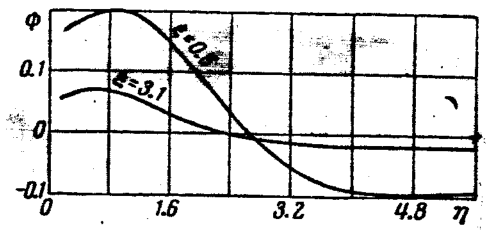

Figure 2.

The results of computation are plotted in the figures. The broken curves are taken from [1].

Figure 1 shows the temperature profiles with respect to the variable η for various values of ξ. With increasing ξ the temperature profile is deformed from a self-similar profile without radiation into one which corresponds to the consideration of the radiation in the approximation of nonlinear (radiant) heat conduction. This deformation proceeds in such a way that the hot gas cools in the presence of radiation while the gas near the wall heats up. On the physical level this means that the hot gas cools more rapidly and the gas near the wall cools more slowly (in comparison with the case where radiation is not considered). The reason for this is that the hot gas surrenders heat not only by way of molecular heat conduction, but by radiation as well, and that the cold gas /79 near the wall absorbs more than it radiates (see Fig. 2 where the dependence on η of the quantity Φ proportional to $- \partial q_r / \partial \eta$ is shown for various values of ξ).

Figure 3 shows the thickness of the boundary layer η_* computed from a value of the temperature which differs from the limiting value by 1%. With small ξ one observes a very abrupt thickening of the boundary layer. Thereafter the thickness of the boundary layer with respect to the variable η remains almost constant and close to its limiting value (for $\xi \to \infty$).

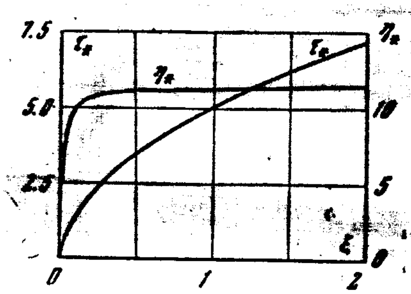

Figure 3.

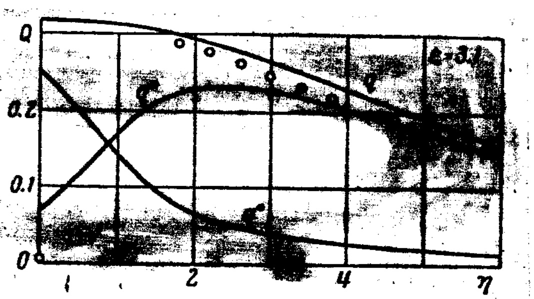

Figure 4.

Under the specified conditions the deformation of the temperature profile with small values of ξ proceeds abruptly due to the rapid increase in the optical thickness of the boundary layer (Fig. 3), terminating sooner in the interior of the flow. This latter fact means that the approximation of nonlinear heat conduction for the radiation first becomes acceptable in the hottest portion of the boundary layer. The same conclusion may be drawn from Fig. 4, where the small circles identify the radiation heat fluxes computed in the nonlinear heat conduction approximation on the basis of the non-selfsimilar temperature profile for ξ = 3.1.

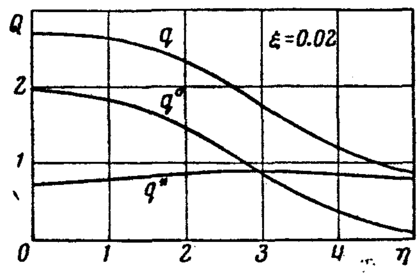

Figure 5.

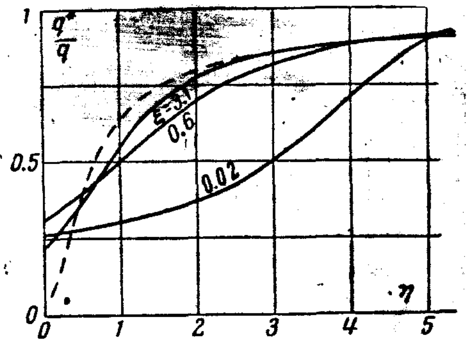

Figure 6.

The region where the temperature profile is nearly linear contracts with increasing ξ in the presence of radiation. This may be seen immediately from Fig. 1 as well as by considering Figs. 4 and 5 which show the fluxes of heat $Q = q/\sigma T_{\infty}^{4}$ due to radiation q^*, ordinary heat conduction q^{o}, and the total q as a function of η for ξ equal to 0.02 and 3.1. The molecular heat flux near the wall diminishes more slowly for small values of ξ and more rapidly for large values of ξ. Since the heat flux due to ordinary heat conduction is proportional to the slope of the temperature profile, the profile curvature increases with increasing ξ.

The radiation flux varies weakly with respect to boundary layer thickness for small values of ξ (Fig. 5), since the boundary layer is still optically transparent (Fig. 3). But with ξ = 0.02 there is already a clearly distinguishable radiation flux maximum due to the fact that heat is originally transferred by radiation, and that it is only near the wall that redistribution occurs with heat being transferred by molecular heat conduction while the total heat flux remains almost constant. This effect is more marked with large ξ. This is clear from Fig. 6, where we see the proportion of the radiation flux as a function of η for various ξ. Within a relatively broad region of the boundary /80 layer the proportion of the radiation flux increases with ξ, rising at an especially high rate for smaller values of ξ. This growth in the radiation contribution to heat transfer is due to the fact that the temperature profile in this region becomes steeper. The situation is reversed near the wall: the steepness of the profile increases and the proportion of radiated heat drops. (For small ξ the relative contribution of the radiation flux increases near the wall as well due to the rapid decline of the molecular flux).

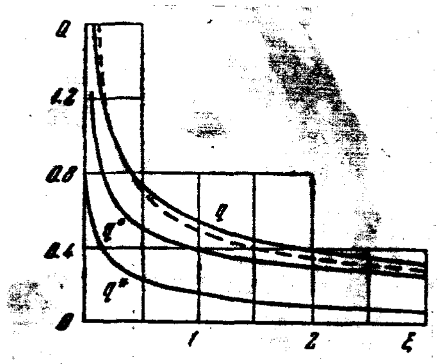

Figure 7.

Figure 7 shows the heat flux to the wall due to radiation, the heat flux due to ordinary heat conduction, as well as the total flux. It also shows the total heat fluxes with consideration of radiation in the nonlinear heat conduction approximation (from [1]). The difference under the conditions in question is somewhat in excess of 10%. The difference in the radiation component of the heat flux is greater (for $\xi = 3.1$, they differ by a factor of 40 under the specified conditions).

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